THEORY OF THE RADIOMETER EFFECT

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A study is reported of the origin of the force exerted on a plate by a Knudsen gas.

In 1873, Crookes used an instrument to measure low gas pressures in the form of a glass vessel having a shaft bearing a cross with thin mica plates attached to the ends, which lay in planes passing through the axis of rotation. Each mica plate was coated on one side with lamp black, while the other was left clean.

If the vessel was evacuated and placed near a source of light or heat, the cross began to rotate; it was afterwards established that rotation began only when the mean free path of the molecules became comparable with the size of the vessel.

This instrument was called a Crookes radiometer, and the effect became called the radiometer effect.

Attempts have been made [1, 2] to extend the theory of the Knudsen manometer [3] to the Crookes radiometer, but it was shown long ago [4] that this approach is unsound.

The force is now explained in terms of differences in the coefficients of reflection for molecules at the blackened and clean surfaces, on the assumption that the temperatures of the two surfaces are equal.

Consider a plate in a low-density gas in equilibrium with the walls of the vessel, which have temperature T_1; let the plate temperature be T_2, while α_1 and α_2 are the reflection coefficients for the clean and blackened sides respectively. The distribution function for the gas particles in the volume takes the form [5]

$$f(\mathrm{p}) = \frac{2m}{\pi}\frac{A}{(2mkT_1)^2}e^{-\frac{p^2}{2mkT_1}}.$$

Then the particle flux on one side of a plate is A.

The particle flux reflected from the blackened side is $\alpha_2 A$; there is also a flux of particles desorbed from this side, which has a distribution

$$f_2(\mathrm{p}) = \frac{2m}{\pi}\frac{A_2}{(2mkT_2)^2}e^{-\frac{p^2}{2mkT_2}}.$$

The flux of these particles is A_2. The gas is in a steady state, so the incident and leading fluxes are equal:

$$A = \alpha_2 A + A_2, \quad A_2 = (1-\alpha_2)A.$$

The distribution for the desorbed particles on the clean side is

$$f_1(\mathrm{p}) = \frac{2m}{\pi}\frac{A_1}{(2mkT_2)^2}e^{-\frac{p^2}{2mkT_2}},$$

and $A_1 = (1-\alpha_1)A$ by analogy with the previous.

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We chose a coordinate system in which the z axis is perpendicular to the plane of the plate. Then the pressure on the blackened side is

$$P_2 = \frac{1}{m} \int_{p_Z<0} p_Z^2 dp f(p) + \frac{\alpha_2}{m} \int_{p_Z>0} p_Z^2 dp f(p) + \frac{1}{m} \int_{p_Z>0} p_Z^2 dp f_2(p).$$

Similarly, the pressure on the clean side is

$$P_1 = \frac{1}{m} \int_{p_Z<0} p_Z^2 dp f(p) + \frac{\alpha_1}{m} \int_{p_Z>0} p_Z^2 dp f(p) + \frac{1}{m} \int_{p_Z>0} p_Z^2 dp f_1(p).$$

In the first term we have integration over the half space of momenta pz < 0 while in the second and third we have the same over the half space pz > 0.

The difference in pressure $\Delta P = P_2 - P_1$

$$\Delta P = \sqrt{\frac{\pi mk}{2}} (\alpha_1 - \alpha_2) [\sqrt{T_2} - \sqrt{T_1}] A.$$

The constant A can be expressed in the gas density N [5]:

$$A = N \sqrt{\frac{kT_1}{2\pi m}}.$$

We have

$$\Delta P = \frac{1}{2} NkT_1 (\alpha_1 - \alpha_2) \left[\sqrt{\frac{T_2}{T_1}} - 1 \right].$$

We can rewrite this expression by replacing NkT_1 by the residual gas pressure P:

$$\Delta P = \frac{1}{2} P (\alpha_1 - \alpha_2) \left[\sqrt{\frac{T_2}{T_1}} - 1 \right]. \quad (1)$$

These formulas show that the Crookes radiometer and the Knudsen radiometric manometer employ different forms of interaction between the gas particles and the solid surface.

We see from (1) that the torque in the Crookes radiometer arises only if two conditions are met simultaneously:

a) the temperature of the plate must differ from that of the vessel walls, and

b) the reflection coefficients from the two surfaces must be different.

In a dense gas, the mean-free path is less than the size of the system, and a temperature difference arises between plate and wall on account of the collisions between the gas molecules. The particles incident on the plate have a thermal velocity corresponding to the gas temperature at a distance about the mean-free path of the plate surface. As the density increases, the mean-free path tends to zero, while the temperature of the incident particles tends to the plate temperature, and the effect vanishes

One can use (1) to determine $\alpha_1 - \alpha_2$ by experiment; in fact, the torque is

$$M = nlS\Delta P,$$

where l is the length of the arm, S is the area of the plate, and n is the number of plates.

We substitute from (1) for ΔP and solve the equation for $\alpha_1 - \alpha_2$ to get

$$\alpha_1 - \alpha_2 = \frac{2M}{lSnP \left[\sqrt{\frac{T_2}{T_1}} - 1 \right]}. \quad (2)$$

If one measures the quantities on the right in (2), one can calculate $\alpha_1 - \alpha_2$.

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LITERATURE CITED

1. W. Heintze, Introduction to Vacuum Technique [Russian translation], Vol. 1, Énergoizdat, Moscow (1960).
2. S. Dushman, The Scientific Principles of Vacuum Technique [Russian translation], Mir, Moscow (1964).
3. M. Knudsen, Ann. Phys., 39, 809 (1910).
4. J. Groszinskii, High-Vacuum Technology [Russian translation], IL, Moscow (1957).
5. V. V. Andreev and L. V. Tanatarov, Ukr. Fiz. Zh., 16, 1953 (1971).